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Technical Notes

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Energy Deposition/Absorption Effects on a Planar Shock Wave

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Nomenclature

e	=	inner energy
L	=	energy deposition/absorption zone length
M	=	Mach number
p	=	pressure
p_0	=	total pressure
$\dot{Q}$, q	=	energy deposition/absorption strength
R	=	universal gas constant
T	=	temperature
u	=	velocity
γ	=	specific heat ratio, 1.4
ρ	=	density
σ	=	deposition/absorption strength parameter, $L\dot{Q}/(RT_1\sqrt{\gamma RT_1})$

Subscripts

1	=	preenergy deposition/absorption state
2	=	postenergy deposition/absorption state
3	=	postshock state

I. Introduction

ENERGY deposition for modification of the external flowfield around a vehicle is a well-known technique and has been studied and used to reduce the drag on the objects flying at supersonic speeds. Since the 1950s, energy deposition in the supersonic flow was proposed as an alternative technique to reduce drag on flying objects [1,2]. Many papers discuss the significant drag decrease by energy deposition in front of an object. Analytical, numerical, and experimental studies were reported [3–11].

Many energy deposition applications share the same flowfield structure: in front of a shock wave, energy is deposited and results in large pressure changes behind the shock wave. This note aims to analytically study a simplified, yet related, one-dimensional problem, which is shown in Fig. 1. In front of a stationary planar normal shock wave, there is a region (with length L) in which energy is deposited or absorbed. There are three regions in the flowfield: the predeposition region 1, the preshock and postdeposition region 2, and the postshock region 3. Between regions 1 and 2, energy is deposited/absorbed with a strength of $\rho\dot{Q}$. The properties in each region can be considered constant. This note aims to obtain the preshock energy deposition/absorption effects on the postshock properties. This problem has applications in external flows, such as the drag reduction for flying objects, and in internal flows, such as supersonic, heated/cooled gas into a combustor.

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II. Analytical Solutions and Discussions

With an aim to obtain analytical solutions, we select the Euler equations and neglect the gas viscosity and thermal conductivity. The one-dimensional governing equations with energy deposition/absorption are [12]

$$p = \rho RT \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(p + \rho u^2)}{\partial x} = 0 \tag{3}$$

$$\frac{\partial(e)}{\partial t} + \frac{\partial(pu + eu)}{\partial x} = \rho\dot{q} \tag{4}$$

The inner energy has the following relation with respect to the other variables: $e = p/(\gamma - 1) + \rho u^2/2$. Note, as stated earlier, the right-hand side of Eq. (4) is zero everywhere in the flowfield, except between regions 1 and 2, where it has a constant value of $\rho\dot{Q}$.

For a steady flow with a stationary planar shock wave, the preceding equations reduce to simple algebraic relations. Especially, the energy equation can be simplified as

$$\frac{\gamma}{\gamma-1}p_1u_1 + \frac{1}{2}\rho_1u_1^3 + \rho_1\dot{Q}L = \frac{\gamma}{\gamma-1}p_2u_2 + \frac{1}{2}\rho_2u_2^3 = \frac{\gamma}{\gamma-1}p_3u_3 + \frac{1}{2}\rho_3u_3^3$$

with an assumption that across the deposition/absorption layer, the density maintains a constant of ρ_1.

We define a nondimensional parameter $\sigma = L\dot{Q}/(RT_1\sqrt{\gamma RT_1})$ to represent the energy deposition/absorption strength. Then, from the algebraic relations, the following simple relation between ρ_1 and ρ_2 exists:

$$\frac{\gamma+1}{2(\gamma-1)}\gamma M_1^2\left(\frac{\rho_1}{\rho_2}\right)^2 - \left(\frac{\gamma}{\gamma-1} + \frac{\gamma^2M^2}{\gamma-1}\right)\frac{\rho_1}{\rho_2} + \left(\frac{\gamma}{\gamma-1} + \frac{\gamma M_1^2}{2} + \frac{\sigma}{M_1}\right) = 0 \tag{5}$$

With the compatible relation that $\rho_1 = \rho_2$ without any energy deposition and absorption, the following root of Eq. (5) is the correct relation between ρ_1 and ρ_2:

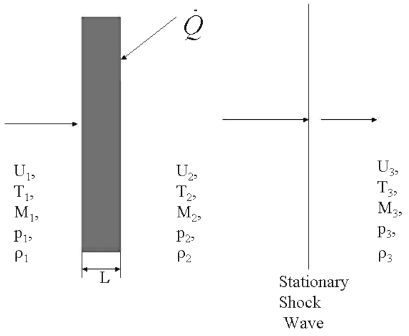

Fig. 1 Illustration of the flow problem.

$$\frac{\rho_1}{\rho_2} = \frac{\gamma + \gamma^2 M_1^2 + A}{(\gamma+1)\gamma M_1^2} \tag{6}$$

where

$$A = \sqrt{(\gamma + \gamma^2 M_1^2)^2 - 2(\gamma/(\gamma-1) + \gamma M_1^2/2 + \sigma/M_1)(\gamma^2-1)\gamma M_1^2}$$

The other relations between regions 1 and 2 are

$$\frac{p_2}{p_1} = \frac{\gamma M_1^2 + 1 - A}{\gamma + 1} \tag{7}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \frac{(\gamma + \gamma^2 M_1^2 + A)}{(\gamma+1)\gamma M_1^2}\frac{(\gamma M_1^2 + 1 - A)}{\gamma + 1} \tag{8}$$

$$\frac{M_2^2}{M_1^2} = \frac{\rho_1}{\rho_2}\frac{p_1}{p_2} = \frac{[\gamma + \gamma^2 M_1^2 + A]}{[\gamma M_1^2][\gamma M_1^2 + 1 - A]} \tag{9}$$

It can be shown that the energy deposition or absorption cannot be infinitely strong; on one limit, A needs to be a real number, and on the other limit, p_2 should be greater than zero. With these two relations, the following relation can be obtained:

$$\sigma_{\min} = \frac{\gamma - 1 - 2\gamma M_1^2}{2(\gamma-1)\gamma M_1} < \sigma < \frac{\gamma(M_1^2-1)^2}{2M_1(\gamma^2-1)} = \sigma_{\max} \tag{10}$$

Note that the scope of energy deposition is much wider than that for energy absorption with $\sigma_{\max} > |\sigma_{\min}|$; when $\sigma = 0$, regions 1 and 2 have the same properties, whereas with the maximum energy deposition, $\rho_2/\rho_1 = (\gamma+1)M_1^2/(1+\gamma M_1^2)$, $p_2/p_1 = (\gamma M_1^2 + 1)/(\gamma+1)$, $T_2/T_1 = (1+\gamma M_1^2)^2/[(\gamma+1)^2 M_1^2]$, and $M_2 = 1$. The maximum energy deposition value is little different from Georgievskii and Levin's [4] previous result, because the definition of σ was different, and a Gaussian distribution for the energy deposition, extending to an infinite distance in front of the shock wave, was assumed in their paper. Equation (10) indicates there is a critical value for the energy absorption situation, as well.

Note that all properties across the deposition/absorption layer are completely determined by the nondimensional parameter σ and the incoming Mach number. When the incoming Mach number becomes large, the postdeposition effects diminish.

With the deposition/absorption strength properly bounded, we can further compare its effect on the postshock properties, which is the major purpose of this note. From region 2 to region 3, the classical Rankine–Hugoniot relations hold, which can be found in many aerodynamics books [13]. The final relations between regions 3 and 1 are

$$\frac{p_3}{p_1} = \left[\frac{2\gamma M_2^2 - (\gamma-1)}{\gamma+1}\right]\left[\frac{\gamma M_1^2 + 1 - A}{\gamma+1}\right] \tag{11}$$

$$\frac{u_3}{u_1} = \frac{\rho_1}{\rho_3} = \frac{\gamma + \gamma^2 M_1^2 + A}{(\gamma+1)\gamma M_1^2}\left[\frac{2 + (\gamma-1)M_2^2}{(\gamma+1)M_2^2}\right] \tag{12}$$

$$\frac{T_3}{T_1} = \left(\frac{[2\gamma M_2^2 - (\gamma-1)][2 + (\gamma-1)M_2^2]}{(\gamma+1)^2 M_2^2}\right) \times \left(\frac{(\gamma + \gamma^2 M_1^2 + A)}{(\gamma+1)\gamma M_1^2}\frac{(\gamma M_1^2 + 1 - A)}{\gamma+1}\right) \tag{13}$$

$$\frac{p_{03}}{p_{01}} = \frac{[(\gamma+1)M_2^2]^{\gamma/(\gamma-1)}}{[\frac{2\gamma M_2^2 - (\gamma-1)}{(\gamma+1)}]^{1/(\gamma-1)}}\frac{1}{(2 + (\gamma-1)M_1^2)^{\gamma/(\gamma-1)}} \times \left(\frac{\gamma M_1^2 + 1 - A}{\gamma+1}\right) \tag{14}$$

To discuss these analytical results, the preshock and postshock conditions vs the deposition/absorption strength are plotted with an incoming Mach number $M_1 = 10.0$ and $\gamma = 1.4$.

Figure 2 shows the energy deposition/absorption effects on the preshock and postshock Mach numbers and densities. When the deposition strength increases, the preshock density increases and the Mach number rapidly decreases, resulting in a weaker shock wave. When the deposition reaches its maximum value, the shock ceases to exist, and this maximum deposition value is a critical point. On the other hand, stronger absorption will decrease the preshock density and increase the Mach number in front of the shock. At the absorption limit, an infinitely large preshock Mach number is created and the postshock density reaches the hypersonic limit of six.

Figure 3 shows the deposition/absorption effects on the static temperatures and postshock stagnation temperature. As the

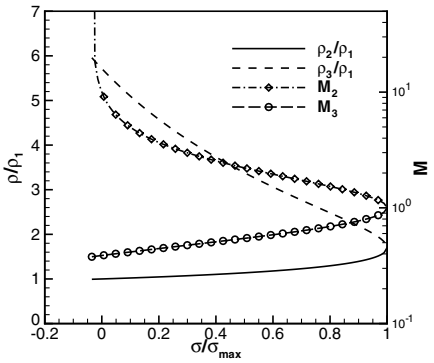

Fig. 2 Energy deposition/absorption effects on Mach number and density results; $M_1 = 10.0$ and $\gamma = 1.4$.

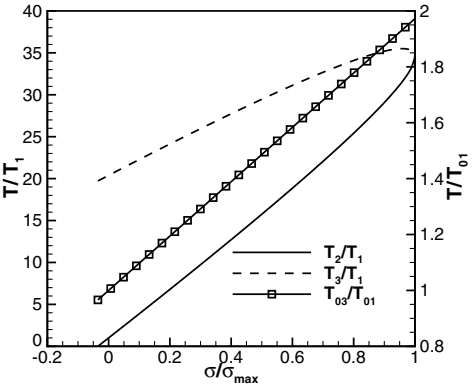

Fig. 3 Energy deposition/absorption effects on temperature results; $M_1 = 10.0$ and $\gamma = 1.4$.

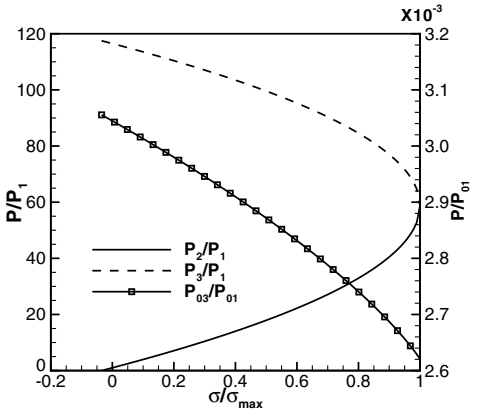

Fig. 4 Energy deposition/absorption effects on pressure results; $M_1 = 10.0$ and $\gamma = 1.4$.

deposition increases, all the temperatures continue to increase; although continuing energy absorption eventually results in a zero static preshock temperature.

Figure 4 shows that stronger energy deposition results in a higher postshock pressure, but lower postshock static and stagnation pressure; although energy absorption will result in higher postshock static and stagnant pressure. We can conclude that if an object exists behind the shock, the heat deposition in front of a shock reduces the wave drag.

III. Conclusions

In this study, we analyzed the effects of energy deposition and absorption on the postshock properties. With Euler Equations, simple algebraic relations exist among the three regions. The results indicate that energy depositions increase the gas density in the postdeposition region and result in lower preshock Mach numbers, and with the critical energy deposition value, the shock ceases to exist. Stronger energy absorption in front of the shock increases the preshock Mach number. Overall, energy depositions result in higher postshock temperature, lower density, and lower static/stagnation pressure, whereas energy absorption yields the exact opposite effects.

Acknowledgments

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